Linear Programs (our first “Oracle”)

Problems written as set of linear inequalities, plus goal to maximize/minimize

\[
\text{goal: maximize } 1000 \text{gold} + 50 \text{silver}
\]

Example: A treasure pile has 20 bars of gold, 1lb each, worth $1,000/bar
400 silver coins, 0.1lb each, worth $50/coin. If you can only carry C lbs of treasure, how much should you take (C is a constant).
For each cell $i$

Position in the floorplan $x_i, y_i$, width $w_i$, height $h_i$

Computed by ILP

\[
\text{Aspect Ratio: } \frac{w_i}{h_i} \quad \text{min. aspect: } \text{min. aspect}_i \quad \text{max. aspect: } \text{max. aspect}_i
\]

\[
\text{Area: } \text{area}_i
\]

For the floorplan

\[
(x_i, y_i) \quad \text{width: } w_i \quad \text{height: } h_i
\]

\[
(F_{\text{width}}, F_{\text{height}}) \quad \text{area: } \text{area}_i \quad \text{computed by ILP}
\]
Rules → Equation

* Cells cannot overlap
* Cells must be inside the floor plan
* Cells within must obey area + aspect requirements

Goal: Minimize the floor plan's area
Cells cannot overlap

Cell \( j \) and cell \( k \)

\[
\begin{align*}
\text{Cell } k \text{ above} & : y_j + h_j \leq y_k \\
\text{Cell } k \text{ to the right} & : x_j + u_j \leq x_k + \infty \cdot (p_{jk} + q_{jk}) \\
\text{Cell } k \text{ below} & : y_k + h_k \leq y_j + \infty \cdot (1 - p_{jk}) \\
\end{align*}
\]

Binary encoding

Integer \( p_{jk}, q_{jk} \)

\[
\begin{align*}
0 \leq p_{jk}, q_{jk} \leq 1 \\
0 \leq q_{jk}, q_{jk} \leq 1 \\
\end{align*}
\]

Code \( p_{jk}, q_{jk} \) computed by MIP
Area for cell R: \( VR, YR, WR, HR \) \( \subseteq \) ILP solver
Input: \( \text{Area}_R, \text{min-aspect}_R, \text{max-aspect}_R \)

Area: \( WR \times HR \geq \text{Area}_R \)

Min-aspect: \( \frac{WR}{HR} \geq \text{min-aspect}_R \)

\( \text{Not linear} \)

\( \text{min-aspect} \left( c, d \right) \)

\( h_R = \text{slope}_R \times W_R + \text{intercept}_R \)

\( h_R \leq \text{max-height}_R \)

\( W_R \leq \text{max-width}_R \)

\( \text{max-aspect} \)

\( a, b \)
Ensure that cell $y_m$ is in the floor plan

$0 \leq x_m \leq y_m$
$y_m + h_m \leq F_{\text{width}}$
$y_m + h_m \leq F_{\text{height}}$
Goal:

\[
\text{Minimize } F_{\text{width}} \times F_{\text{height}}. \quad \text{Not linear}
\]

Pick an \( F_{\text{width}} \), put in as a constant + Run

\[
\text{Minimize } 1300 \times F_{\text{height}}
\]
Mixed Integer Linear Programming for Floorplanning

Previous techniques use relatively local optimization to form tree
Develop more global solver technique

Use Integer Linear Programming (ILP)
Formulate as series of linear integer equations and an optimization goal:

1.) No cells can overlap
2.) Cells must be given the proper area
3.) Minimize the overall area of the floorplan

ILP is NP-Complete, but good heuristic algorithms are available
Cell Variables

For cell i

\( \text{min\_aspect}_i, \text{max\_aspect}_i \): provided minimum & maximum aspect ratio \( w_i/h_i \)

\( \text{area}_i \): provided required area

\( w_i, h_i \): calculated width and height

\( (x_i, y_i) \): calculated coordinates of lower left corner

For floorplan

\( \text{F\_width}, \text{F\_height} \): calculated overall floorplan width and height

\( \text{min\_aspect}_i \leq (w_i/h_i) \leq \text{max\_aspect}_i \)
Non-Overlapping Cells

To not overlap, other cells must be to the right, left, above, or below this one.

\[ y_i + h_i \leq y_k \]
\[ x_k + w_k \leq x_i \]
\[ x_i + w_i \leq x_k \]
\[ y_k + h_k \leq y_i \]

Requires OR of 4 equations

Add binary values \( P_{ik} \) and \( Q_{ik} \), where (00→right, 01→left, 10→up, 11→down)

\( P_{ik} \) integer; \( Q_{ik} \) integer;
\( P_{ik} > -1, P_{ik} < 2, Q_{ik} > -1, Q_{ik} < 2 \)

right(00): \( x_i + w_i \leq x_k + \infty \cdot (P_{ik} + Q_{ik}) \)
left(01): \( x_k + w_k \leq x_i + \infty \cdot (P_{ik} + (1 - Q_{ik})) \)
up(10): \( y_i + h_i \leq y_k + \infty \cdot ((1 - P_{ik}) + Q_{ik}) \)
down(11): \( y_k + h_k \leq y_i + \infty \cdot ((1 - P_{ik}) + (1 - Q_{ik})) \)
Cell Area & Aspect Ratios

Problem:
Area is non-linear in width and height

Solution:
Linearize the function (inexact)

\[ h_i = \text{slope}_i \times w_i + \text{intercept}_i \]

Aspect ratio:
Given area and aspect ratio, can determine
max_width and max_height

\[ h_i \leq \text{max_height}_i \]
\[ w_i \leq \text{max_width}_i \]
Minimize Overall Area

Goal is "Minimize the overall area of the floorplan"

Again, area is not a linear equation

Solution:

Fix overall width, and solve for height. Binary search width for best area

\[ \text{minimize } F_{\text{height}} \]

\[ y_i + h_i \leq F_{\text{height}} \]

\[ x_i + w_i \leq F_{\text{width}} \]

\[ 0 \leq y_i \]

\[ 0 \leq x_i \]

Cell inside floorplan