Using the FFT

In this lab you will learn some of the more important relationships between a sequence and its DFT. We will concentrate on a few of the relationships between operations in the time-domain and corresponding frequency-domain operations.

1. **DTFT vs. DFT**

   In this problem you will observe the errors that are produced when applying the DFT to infinite duration sequences. Let $h[n] = a^n u[n]$ with $a = 0.75$. Its DTFT is: $H(\Omega) = \frac{1}{1-ae^{-j\Omega}}$. Generate the 16-point DFT sequence H[k] by evaluating $H(\Omega)$ at the 16 appropriate frequency points. Form $h_{I16}[n]$, the 16-point IDFT of $H[k]$ and the error sequence, $e_{I16}[n] = h[n] - h_{I16}[n]$. Repeat for $N = 32$. Turn in a plot of $h_{I16}[n]$ and $e_{I16}[n]$ on one page using subplot. Repeat for $N = 32$. Describe the differences between the inverse DFTs of the 16-point and 32-point cases.

2. **Performing Convolution with the DFT**

   In many situations we want to perform the time domain operation of convolution in the frequency domain. If we multiply two DFTs, this corresponds to *circular* convolution in the time domain. We can still multiply two DFTs to do a linear convolution in time if we choose the length of the DFT to be long enough. We'll investigate this here. Say we want to compute the linear convolution $y[n] = h[n] * x[n]$ where $x[n] = h[n] = u[n] - u[n-7]$.

   Let $N = 16$ where N is the number of DFT points. We are picking $N = 16$ since it is a power of two (so that Matlab uses an FFT) and to avoid wrapping around in the circular convolution of $x[n]$ and $h[n]$. Therefore $x[n]$ and $h[n]$ should each have length 16. Hand in plots of the following sequences for the case of 16 DFT points. Plot the results on the same page whenever possible, (i.e. use SUBPLOT).

   (a) The original time sequence x[n].

   (b) The magnitude spectrum of $X[k]$, the DFT of $x[n]$.

   (c) The magnitude spectrum of the product of the two DFT sequences $X[k]$ and $H[k]$.

   (d) The Inverse DFT sequence.

   (e) The actual $y[n]$ you would get if you did the direct convolution $y[n] = x[n] * h[n]$.

   Lastly, explain what happens when you choose the number of DFT points to be too small (Let $N = 8$ and repeat steps (a) - (e) BUT DON'T HAND IN ANY PLOTS).

3. **Interpolation in the Frequency Domain**

If we have N samples in time, we can only get N DFT points. It is possible to compute the DFT at a denser set of points in frequency by padding the time domain sequence with zeros. Let

$$x[n] = r^n cos(\Omega_0 n) \quad \text{for } 0 \leq n \leq 63$$

with $r = 0.7$ and $\Omega_0 = 9\pi/64$. Compute the 64-point DFT $X[k]$ and plot the 33-points of the magnitude spectrum corresponding to the range $0 \leq \Omega \leq \pi$. Generate the 128-point sequence $y[n]$ by padding $x[n]$ with zeros:

$$y[n] = \begin{cases} x[n] & \text{for } \ 0 \leq n \leq 63 \\ 0 & \text{for } 64 \leq n \leq 127. \end{cases}$$

Compute the 128-point DFT $Y[k]$ and plot the 65 points of the magnitude spectrum corresponding to $0 \leq \Omega \leq \pi$. Compare the plots. Print and hand in plots of the two magnitude spectra.

4. **Interpolation in the Time Domain**

If you want more samples in time, time-domain interpolation can be achieved by zero-padding the frequency domain function. Let $x[n] = cos(\pi n/16)$, for $0 \leq n \leq 31$.

(a) Plot the sequence $x[n]$.

(b) Compute the 32-point DFT of $x[n]$ and using the matlab functions *real* and *imag*, plot the real and imaginary sequences $X_R[k]$ and $X_I[k]$ for $0 \leq k \leq 31$.

(c) Remembering that matlab indexes its vectors from 1 (not 0), pad the sequence $X[k]$ with 32 zeros to produce $Y[k]$. The zero-padding is performed in the high frequencies as follows:

$$Y[k] = \begin{cases} X[k] & for \ \ 0 \leq k \leq 15 \\ 0 & for \ 16 \leq k \leq 47 \\ X[k-32] & for \ 48 \leq k \leq 63. \end{cases}$$

Plot $Y_R[k]$ and $Y_I[k]$, the real and imaginary parts of $Y[k]$, respectively.

(d) Compute $y[n]$, the 64-point IDFT of $Y[k]$. Plot the real and imaginary components of $y[n]$ Compare $y[n]$ with with $x[n]$. Print and hand in plots of $x[n]$ and the real and imaginary parts of $y[n]$).