Fourier Series

In this lab, we will study the truncated Fourier series reconstruction of the periodic function $x(t)$ shown in figure 1.

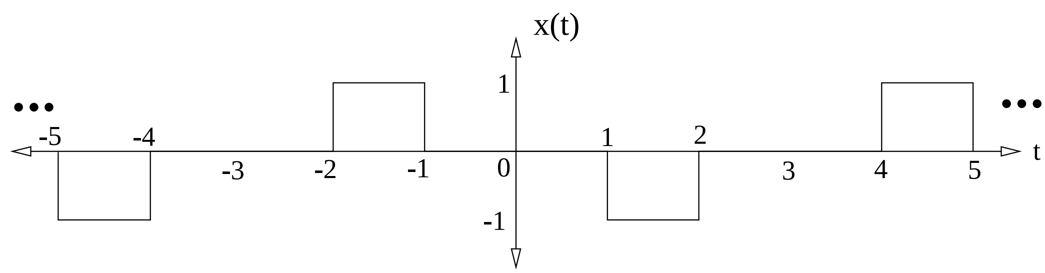

Figure 1: Periodic Signal

1. The equations required to do this lab are thoroughly covered in the textbook, Sections 4.2 – 4.3.

2. Compute the Fourier Series coefficients C_k for $x(t)$. Do this mathematically (i.e., using calculus, not MATLAB) using the Fourier series analysis equation

$$C_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} dt \tag{1}$$

   where T_0 is the fundamental period of $x(t)$ and $\omega_0 = 2\pi/T_0$. Your derivation of the coefficients C_k should be turned in as part of your lab write-up.

   For reference, you should obtain as your answer

$$C_k = \begin{cases} 0 & \text{if } k = 0,\ k \text{ even} \\ \frac{1}{jk\pi}[\cos(2k\pi/3) - cos(k\pi/3)] & \text{if } k \text{ odd} \end{cases}$$

   Hint: Check carefully that you have the correct value for T_0. (By inspection from figure 1, $T_0 = 6$.)

3. Using MATLAB, plot the magnitude and phase spectrum of C_k for $k \in \{-10, \ldots, 10\}$ (see section 4.3 in the textbook). Use the MATLAB command `stem` instead of `plot` to emphasize that the spectra are *line* spectra and not a continuous function of k. Label your axes and title your graphs.

   Hints: You will need the following MATLAB commands:

   - `abs` to compute $|C_k|$.
   - `phase` to compute the phase angle of C_k.

- You may wish to write a MATLAB function to compute C_k for any k – this will be useful in section 4.

4. We know that the original signal $x(t)$ can be recovered exactly from the Fourier series coefficients C_k using the synthesis equation

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_o t} \tag{2}$$

   where C_k are the coefficients computed using eq. 1.

   In this lab, we wish to study the effect on the reconstructed signal of using only a *finite* number of terms in the synthesis equation. Call this (distorted) signal $\hat{x}(t)$ and compute it, for a given $k_{\max}$, as

$$\hat{x}(t) = \sum_{k=-k_{\max}}^{k_{\max}} C_k e^{jk\omega_0 t} \tag{3}$$

   This equation differs from the exact synthesis equation only in that the sum is now over a finite number of terms. The coefficients C_k are unchanged.

   Using MATLAB, write whatever script/function files you feel are appropriate to implement the calculation of $\hat{x}(t)$ in eq. 3. Produce plots of $\hat{x}(t)$ for $t \in [-4, 4]$ for each of the following cases:

   (a) $k_{\max} = 5$
   (b) $k_{\max} = 15$
   (c) $k_{\max} = 25$

   For all the plots, use as your time-axis t the MATLAB vector

```
>> t=-4:.01:4;
```

   Label all axes and title your plots.

5. Your write-up should include the following:

   (a) Derivation of the coefficients C_k.
   (b) Spectra of C_k as two plots: $|C_k|$ and the phase angle of C_k.
   (c) The three plots of truncated reconstructions of $\hat{x}(t)$ from section 4.
   (d) The script/function files that you wrote for section 4.