The S-plane

The purpose of this MATLAB assignment is to gain a better understanding of the S-plane and filters. Specifically, you will be examining the behavior of first and second order high-pass and low-pass filters of the form $H(s) = \frac{N(s)}{D(s)}$ where $N(s)$ and $D(s)$ are polynomials in terms of s. An nth order filter of this form will have n poles and n zeros any of which may or may not be finite. To test the function of your filter $H(s)$, compute $H(0)$ and $H(\infty)$. If $H(0) = 0$ and $0 < H(\infty) < \infty$, then $H(s)$ is a high-pass filter. If $0 < H(0) < \infty$ and $H(\infty) = 0$, then $H(s)$ is a low-pass filter. If $H(0) = 0$ and $H(\infty) = 0$, then $H(s)$ is a band-pass filter. If $0 < H(0) < \infty$ and $0 < H(\infty) < \infty$, then $H(s)$ is a band-reject filter.

Download the m-file, `frevals.m`, from the web page for this assignment. The m-file `frevals.m` determines the frequency and impulse response of any system function where the system function is defined by finite pole and zero locations. By typing HELP `frevals` in MATLAB, an explanation of the proper syntax of the function wil be displayed.

The function operates on three input vectors. The first vector contains the interval, in decades, over which the frequency response of the system will be evaluated. For example, if `[-1 2]` is the first input vector, the frequency response will be plotted (in log-log scale) over the frequencies range 10^{-1} Hz to 10^{2} Hz. Note that this vector may have to be changed to observe the interesting regions of the frequency response of different system functions.

The other two input vectors contain the finite pole and finite zero locations of the desired system. Any input complex values must be entered without spaces, (e.g. `2+.3*j` is a valid entry but `-2 + .3*j` is not). If the system does not contain any finite poles, the null vector `[]` must be used. If the system does not contain any finite zeros, the null vector must be used as well. Keep in mind that a system that has no finite zeros (or poles) may still have a number of zeros (or poles) at infinity.

When doing the following, you should be able to draw general conclusions by varying the parameters of the filter in several experiments and observing changes in the responses due to the changed parameter. This may be done by identifying the variable parameters, and then changing the parameter over a wide range of values where the range consists of the possible values that the parameter may assume. For example, if the system has one parameter which is a pole at $s = x$, examine the behavior of the system for x small and x

large. If the system has two parameters x and y, examine the system when x is equal to y, when x is much larger than y, when x is much less than y, and any other comparison you think appropriate. (Note: s is not a parameter, the pole and zero locations are the parameters).

1. **First Order System**

   (a) Create a first-order low pass system. This consists of a single finite non-zero pole on the real axis. Vary the location of the pole and comment in the changes in the frequency and impulse responses.

   (b) Repeat part (a) for a first-order high pass system. Again, vary the location of the pole and comment on the changes in the frequency and impulse responses.

   You are not asked to hand in any plots in problem 1, but you should list any conclusions about the effects of pole placement on the frequency and impulse responses. You may wish to comment on the effect that pole placement has on the dc-gain (i.e. the value of $H(0)$), the cut-off frequency (i.e. the 3 db half-power point), and the rolloff rate (i.e. how quickly the frequency response falls or rises, a useful measure is db per decade of frequency).

2. **Second Order System**

   (a) Create a system function with impulse response, $h(t) = e^{-\alpha t} sin(\omega_0 t)u(t)$. How do α and ω_0 affect the frequency and impulse responses of the system? Pick α and ω_0 such that the system is obviously a low pass filter. Print the plot containing the magnitude and impulse responses and comment on the information they contain. Make sure you specify the α and ω_0 you used for your system.

   (b) Create a second-order high pass system. Print the plot containing the magnitude and impulse responses. Where did you place the poles and zeros to get the high pass system?