

Some Useful Summations

1. Geometric Progression

$$\sum_{i=0}^{\infty} a^i = \frac{1}{1-a}$$
$$|a| < 1$$

2.

$$\sum_{i=0}^N a^i = \begin{cases} 0 & \text{if } a = 0 \\ N + 1 & \text{if } a = 1 \\ \frac{1-a^{N+1}}{1-a} & \text{otherwise} \end{cases}$$

3.

$$\sum_{i=0}^{\infty} i a^i = \frac{a}{(1-a)^2}$$
$$|a| < 1$$

(Take the derivative in Example 1 above)

4.

$$\sum_{i=1}^N i = \frac{N(N+1)}{2}$$

5.

$$\sum_{i=1}^N i^2 = \frac{N(N+1)(2N+1)}{6}$$

6. Sinc Function

$$\text{sinc } x = \frac{\sin x}{x}$$

7. Taylor Series Expansion of an Exponential

$$e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!}$$